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Two-Point Descriptions of Wake Turbulence with Application to Noise Prediction

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Measurements of the four-dimensional two-point correlation tensor of a fully developed airfoil wake are presented. These data allow examination of the characteristic eddies of the turbulence through proper orthogonal decomposition and linear stochastic estimation. A simple generic technique has been developed to extrapolate the correlation tensor function from the Reynolds stress field based on the hypothesis that its form is largely determined by the constraints imposed by inhomogeneity and continuity. Estimates for the plane wake compare favorably with measurements, and, interestingly, proper orthogonal modes and characteristic eddy structures inferred from the estimates are similar to those obtained from measurements. This implies that these measures of the correlation function are largely determined by fairly basic physical information, suggesting some simplification of the turbulence-modeling problem for applications such as aeroacoustics.

Introduction

THE motivation behind the present work is the desire for more realistic representations of turbulence suitable for use in computing broadband noise resulting from blade–wake interactions in helicopter rotors, stator–wake interactions in aircraft engines, and similar situations. Such calculations (for example, see Amiet[1] and Glegg et al.[2]) require the two-point space–time correlation of the upwash velocity component seen by the lifting surface. Because this information is not generally available, the turbulence is usually assumed to behave as though it were homogeneous and isotropic, and von Kármán's interpolation formula or a similar function is used. In fact, the upwash correlation function in wake flows is quite different from that implied by von Kármán's interpolation; for example, see Devenport et al.,[3] Wittmer et al.,[4] and Wenger et al.[5] It is dominated by the anisotropy and inhomogeneity visible in the mean flow and its expression in the large-eddy structure of the turbulence. Devenport et al.[3] show that these differences can have a significant effect on the broadband noise generated.

To provide a more realistic representation of the correlation function, we have measured the complete two-point space–time correlation tensor for a fully developed airfoil wake. These data have been reduced to a form that we anticipate may be used directly in aeroacoustic calculations. At the same time we have developed a modeling technique that enables us to estimate the two-point correlation tensor function from the Reynolds stress field, such techniques are needed to couple aeroacoustic calculations to Reynolds-averaged Navier–Stokes calculations.

In this paper, we present and compare the measurements and model predictions, concentrating on those features of the instantaneous wake structure that can supposedly be inferred from the two-point correlation tensor. In particular, we use the techniques of linear stochastic estimation and proper orthogonal decomposition to extract three-dimensional representations of the eddy structures. Interestingly, despite the simplicity of our modeling technique, we find that it reproduces many of the gross features of the correlation tensor function. Furthermore, proper orthogonal modes and characteristic eddy structures inferred from the model bear close similarity to those obtained from measurements. This implies that these measures of the correlation function are largely determined by fairly basic physical information, suggesting some simplification of the turbulence-modeling problem for aeroacoustics.

Measurements

Measurements were made in the Virginia Tech Low-Speed Wind Tunnel (Fig. 1). The empty test section of this tunnel, 610 mm high and 914 mm wide, produces a low-turbulence (<0.3%), closely uniform flow with near-zero streamwise pressure gradient. A 203-mm-chord NACA 0012 airfoil was mounted at the midheight of the test section, at zero sweep and angle of attack. A distributed roughness trip, consisting of a single layer of 0.5-mm-diam glass welding beads covering the first 40% of the chordlength of the airfoil, was used to increase the Reynolds number of the airfoil boundary layers and, thus, its wake.

Single- and two-point turbulence measurements were made in the wake of the airfoil using Auspex Corporation Kovaznay-type four-sensor hot-wire probes.[6] These probes are capable of simultaneous three-component measurement from a 0.5-mm^3 measurement volume. They were calibrated directly for flow angle using the method of Wittmer et al.,[6,7] the calibration method having been tested and verified through fully developed turbulent pipe flow measurements. The amplitude and phase response of the separate sensor channels of each probe were optimized, matched, and measured by stimulating their impulse response using a pulsed YAG laser. Flat amplitude and matched phase response characteristics were obtained out to well beyond 20 kHz, more than sufficient for the present study.

A two-axis computer-controlled traverse gear was used to position one of the four-sensor probes in the test section. For the two-point measurements, the second four-sensor probe was held using an unmotorized support. Previous studies by Miranda[8] suggest that interference between the probes would have been minimal. This conclusion is supported by the present results.

Measurements were made for an approach freestream velocity U_e of 27.5 m/s, corresponding to a chord Reynolds number of 3.28×10^5. Cross-sectional mean velocity and turbulence measurements made 8.33 chord lengths downstream of the foil (Fig. 1) showed the wake to be closely two dimensional. Mean velocity, turbulence stress, and triple product profiles measured at 18 stations between $x/c = 0.6$ and 11 showed the wake to have a momentum

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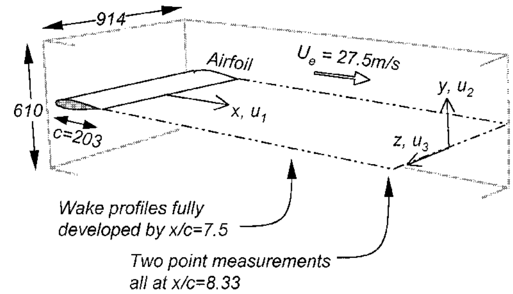

Fig. 1 Schematic of the airfoil and test section showing the coordinate system and location of two-point measurements; dimensions in millimeters.

thickness Reynolds number of 3.06×10^3 and to reach a fully developed state at about $x/c = 7.5$ and $x/\theta = 800$, with self-similarity in all Reynolds stress and triple product profiles.

Two-point measurements were made at $x/c = 8.33$ and $x/\theta = 900$, where the centerline mean velocity deficit U_w was $5.6\% U_e$ and the half-wake width L (distance from the centerline to the location, where the mean velocity deficit is $0.5U_w$) was 17.4 mm. One of the probes (the fixed probe) was placed at 17 positions arranged in a profile from $y/L = 0$ to 3, where y is the distance normal to the plane of the wake, measured from its centerline. For each of these positions, the second moveable probe was placed at some 400 points, each corresponding to a different z and y separation between the probes (up to a maximum of about $4L$). Measurements were made for both positive and negative y separations but only for negative z separations because the correlation function was expected to be symmetric about $z = 0$. Points were taken at a greater density for smaller separations in anticipation of the shape of the correlation function. The smallest distance between the probes was 2.5 mm, about 16 times the expected Kolmogorov length scale, estimated assuming a dissipation rate equal to the maximum rate of turbulence kinetic energy production implied by the measured Reynolds stress and mean-velocity profiles. At each point 50 records of 3072 samples were recorded from each of the eight sensors at 50 kHz, sufficient to calculate a low uncertainty estimate of the cross-spectrum tensor. The two-point cross-spectral estimates were inverse Fourier transformed to obtain estimates of the time-delay correlation for each point pair. These data encapsulate all of the empirical information gathered about the two-point correlation tensor, but do not form a convenient or usable expression of that function. (The size of the reduced data set is some 500 MB.) For this reason, the data were interpolated onto a more easily referenced four-dimensional grid. Care was taken to ensure that the interpolated data contained a faithful representation of the original points. The size of the interpolated correlation tensor function is about 68 MB. These data are available from the first author (see URL: http://www.aoe.vt.edu/flowdata) as an explicit MATLAB function.

The measured space–time correlation $R_{ij} = R_{ij}(y, y', z' - z, \tau)$ depends on the y locations of the two points, the z distance between them, and the time delay τ. It does not depend on absolute z or time because the average flow is homogeneous in these directions. Note that x-wise correlation measurements were not performed. Uncertainty in the correlation measurements, estimated using methods derived from those of Kline and McClintock,[9] was found to be 3.5% at 20:1 odds. Further details of the apparatus and instrumentation, along with discussion of all of the single- and two-point measurements made in the wake are given by Devenport et al.[10] in a report (see URL: http://www.aoe.vt.edu/flowdata).

Modeling

The objective of the modeling effort was to develop a simple technique for extrapolating the two-point correlation tensor function from single-point Reynolds stress data. We were inspired by the need for such models in aeroacoustics, by the smooth and relatively simple form revealed by the two-point correlation tensor measurements, and by the hypothesis that much of this form might be implicit in the inhomogeneous form of the Reynolds stress field coupled with the constraints of continuity. Our philosophy was to develop the simplest possible generic method consistent with this hypothesis. The model is designed to predict the spatial correlation tensor function

$$\Re_{ij}(\boldsymbol{r}, \boldsymbol{r}') \equiv \overline{u_i(\boldsymbol{r}) u_j(\boldsymbol{r}')} \tag{1}$$

where $\boldsymbol{r} = x_i = (x, y, z)$ and $\boldsymbol{r}' = x_i' = (x', y', z')$ denote the two points and $u_i\,(i = 1, 2, 3)$ denotes the fluctuating velocity components in the directions of x, y, and z (see Fig. 1). In incompressible flow, $\Re_{ij}$ must be divergence free with respect to both positions, a property that can be guaranteed[11] by writing it as the double curl of another function, namely, the vector-potential correlation q_{ij},

$$\Re_{ij}(\boldsymbol{r}, \boldsymbol{r}') = \varepsilon_{ikl}\varepsilon_{jmn} \frac{\partial^2 q_{ln}(\boldsymbol{r}, \boldsymbol{r}')}{\partial x_m' \partial x_k} \tag{2}$$

It is straightforward to show that, for homogeneous turbulence,

$$q_{ij}(\boldsymbol{r}, \boldsymbol{r}') = -\tfrac{1}{2} u^2 h(s)\, \delta_{ij} \tag{3}$$

where u is the turbulence intensity, s is the scalar distance between $\boldsymbol{r}$ and $\boldsymbol{r}'$, and $h(s)$ is the first moment of the longitudinal correlation coefficient function, that is,

$$h(s) = \int_0^s s' f(s')\, \mathrm{d}s'$$

[See Hinze[12] for the definition of $f(s)$.] A von Kármán turbulence spectrum (see Ref. 12) implies

$$h(s) = 2^{\frac{2}{3}} \left[\sqrt[3]{2}\,\Gamma\left(\tfrac{4}{3}\right) - (k_e s)^{\frac{4}{3}} K_{\frac{4}{3}}(k_e s) \right] \Big/ \Gamma\left(\tfrac{1}{3}\right) \Big/ k_e^2 \tag{4}$$

where $k_e \approx 0.75/l$ and l is the scale of the turbulence.

For inhomogeneous turbulence we propose an expression analogous to Eq. (3) in which q_{ij} is separated into the product of the homogeneous function $h(s)$ that controls the manner in which the correlation falls off with separation between points and a scaling function $a_{ij}(\boldsymbol{r}, \boldsymbol{r}')$, that is,

$$q_{ij}(\boldsymbol{r}, \boldsymbol{r}') = a_{ij}(\boldsymbol{r}, \boldsymbol{r}') h(s) \tag{5}$$

Substituting Eq. (5) into Eq. (2), we obtain

$$\Re_{ij}(\boldsymbol{r}, \boldsymbol{r}') = \varepsilon_{ikl}\varepsilon_{jmn} \frac{\partial^2 a_{ln}(\boldsymbol{r}, \boldsymbol{r}') h(s)}{\partial x_m' \partial x_k} = \varepsilon_{ikl}\varepsilon_{jmn} \left[\frac{\partial^2 a_{ln}(\boldsymbol{r}, \boldsymbol{r}')}{\partial x_m' \partial x_k} h(s) + \frac{\partial a_{ln}(\boldsymbol{r}, \boldsymbol{r}')}{\partial x_m'} \frac{\partial h(s)}{\partial x_k} + \frac{\partial a_{ln}(\boldsymbol{r}, \boldsymbol{r}')}{\partial x_k} \frac{\partial h(s)}{\partial x_m'} + a_{ln}(\boldsymbol{r}, \boldsymbol{r}') \frac{\partial^2 h(s)}{\partial x_m' \partial x_k} \right] \tag{6}$$

The scaling function a_{ij} is chosen to make the two-point correlation function consistent with the prescribed Reynolds stress field of the flow, that is, when $\boldsymbol{r} = \boldsymbol{r}'$ (when $s = 0$), $\Re_{ij}$ must be equal to the Reynolds stress tensor τ_{ij}. To find a form for a_{ij} that will achieve this we first note that the value of $h(s)$ and its first derivative tend to zero as s tends to zero because it is the first moment of a correlation coefficient function. Also, its second derivative,

$$\frac{\partial^2 h(s)}{\partial x_m' \partial x_k}$$

tends to $-\delta_{mk}$. Thus, for $s = 0$,

$$\Re_{ij}(\boldsymbol{r}, \boldsymbol{r}) = \tau_{ij}(\boldsymbol{r}) = -\varepsilon_{ikl}\varepsilon_{jmn} a_{ln}(\boldsymbol{r}, \boldsymbol{r}) \delta_{mk} = a_{ij}(\boldsymbol{r}, \boldsymbol{r}) - \delta_{ij} a_{oo}(\boldsymbol{r}, \boldsymbol{r}) \tag{7}$$

This equation is satisfied if

$$a_{ij}(\boldsymbol{r}, \boldsymbol{r}) = \tau_{ij}(\boldsymbol{r}) - \tfrac{1}{2}\delta_{ij}\tau_{pp}(\boldsymbol{r}) \tag{8}$$

Given the otherwise second-order differential relationship between the vector-potential and velocity correlation functions, this simple result is unexpected and greatly advantageous. It shows that it is a fairly trivial matter to prescribe a vector-potential correlation function consistent with any Reynolds stress field.

A simple prescription for $a_{ij}(\boldsymbol{r}, \boldsymbol{r}')$ consistent with Eq. (8) can be formed from the average of the Reynolds stress tensors at $\boldsymbol{r}$ and $\boldsymbol{r}'$, that is,

$$a_{ij}(\boldsymbol{r},\boldsymbol{r}') = \tfrac{1}{2}\left[\tau_{ij}(\boldsymbol{r}) - \tfrac{1}{2}\delta_{ij}\tau_{pp}(\boldsymbol{r}) + \tau_{ij}(\boldsymbol{r}') - \tfrac{1}{2}\delta_{ij}\tau_{pp}(\boldsymbol{r}')\right] \quad (9)$$

This formulation has the attractive feature that it guarantees decay of the correlation function at large s (Ref. 13).

To apply this model to a flow requires as input only the Reynolds stress field $\tau_{ij}(\boldsymbol{r})$ and the single constant length scale l, required to scale the decay function in Eq. (4). In effect, an inhomogeneous flow is treated as homogeneous turbulence, modulated to match the Reynolds stress field. However, because the modulation is performed on q_{ij}, the result is not a trivial product of the von Kármán spectrum and the turbulence stress field but a more complex function whose form, and possibly the length scales it implies, vary from point to point in a manner implied by continuity and the inhomogeneity, as well as by the von Kármán form.

To calculate the two-point correlations for the airfoil wake, we specified the Reynolds stress field from the measured profiles at $x/c = 8.33$ and chose the length scale $l = 0.73L$. This value of l was chosen by comparison with the measured time delay correlation on the wake centerline.

Results and Discussion

In this section we present and compare measurements and predictions. In making several of these comparisons Taylor's hypothesis is assumed, that is, we assume the measured space–time correlation function and predicted spatial correlation function may be related as $\Re_{ij}(\boldsymbol{r},\boldsymbol{r}') = R_{ij}(y, y', z'-z, -(x'-x)/U_e)$ and, more generally, that time delay and streamwise separation are interchangeable, that is, $x' - x \equiv -\tau U_e$. We expect Taylor's hypothesis to be reasonably accurate because turbulence intensities in the wake and mean velocity variations across it were fairly small (about 2 and 5%, respectively) at the measurement location.

Correlations and Length Scales

Figure 2 compares measured and modeled correlation maps for zero time delay and zero spanwise separation $R_{ij}(y, y', 0, 0)$. The plots show all nine components of the correlation function. The lines $y = y'$ represent the Reynolds stress profiles.

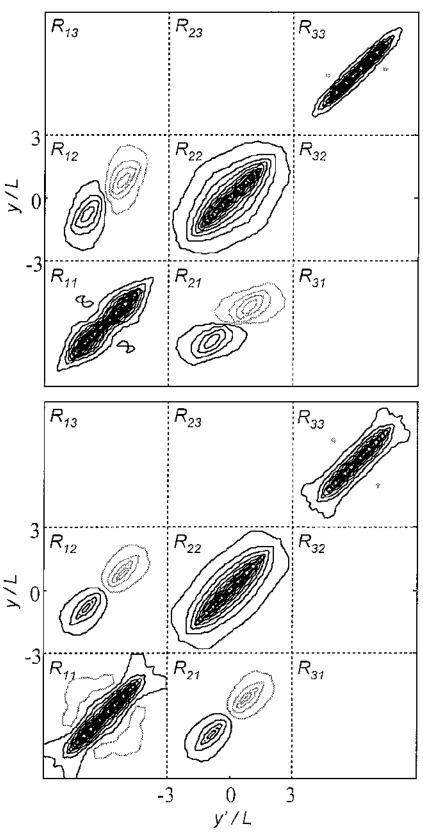

Fig. 2 Measured (top) and modeled (bottom) correlation maps for zero x and z separation $R_{ij}(y, y', 0, 0)/U_w^2$: black lines are positive contour levels in steps of 0.01 from 0.005; gray lines are negative levels in steps of 0.01 down from − 0.005.

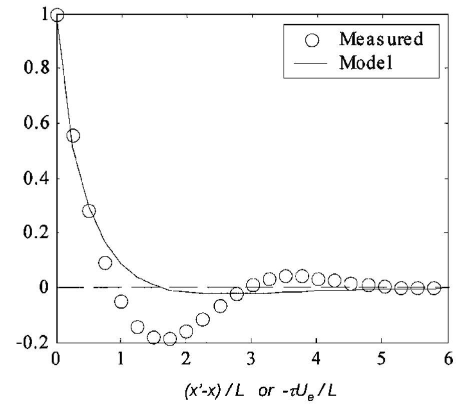

Fig. 3 Time-delay/x-wise correlation coefficient function for u_2 fluctuations on the wake centerline ($y = 0$).

The measured data are fairly symmetric about the bottom left to top right diagonal, an indication that they are closely consistent with the requirement that $R_{ij}(y, y', 0, 0) = R_{ji}(y', y, 0, 0)$. Most of the contours of the normal correlations R_{11}, R_{22}, and R_{33} are eye-shaped, indicating correlations over greater y distances near the wake centerline than at its edges. Not surprisingly, correlations of the vertical velocity component R_{22} extend over the greatest vertical distance, those of the spanwise velocity fluctuations R_{33} over the smallest distance. There are also some more subtle features that may be of significance, namely, wings in the R_{11} correlation near the wake center and asymmetry in the R_{12} correlation associated with the dominant Reynolds shear stress.

Many of the gross features of the correlations are reproduced by the model. The model is by no means perfect, however, and there are also a number of differences, most notably in the extent of the R_{22} correlation and in the predicted symmetry of R_{12}. Perhaps the most significant difference between model and measurement is in the time-delay/x-wise correlation of v fluctuations (Fig. 3). The measured correlation function is oscillatory, presumably as a result of quasi-periodicity of the large-eddy structure. This feature is absent from the model because it includes no such physics.

Another comparison of the measured and model correlations is shown in Fig. 4 in terms of the profiles of local integral length scales. For the jth velocity component we define integral scales as

$$\Lambda_{jx}(y) = U_e \int_0^\infty \frac{R_{jj}(y, y, 0, \tau)}{R_{jj}(y, y, 0, 0)}\,\mathrm{d}\tau$$

$$\Lambda_{jz}(y) = \int_0^\infty \frac{R_{jj}(y, y, z'-z, 0)}{R_{jj}(y, y, 0, 0)}\,\mathrm{d}(z'-z) \quad (10)$$

(no summation implied). In homogeneous turbulence one would expect $\Lambda_{1x} = \Lambda_{3z} = 2\Lambda_{3x} = 2\Lambda_{1z} = 2\Lambda_{2x} = 2\Lambda_{2z}$. Although the measured length scales (Fig. 4) are fairly uniform across the wake, they depart markedly from this relationship. In particular, the lateral integral scale Λ_{2z} is considerably larger than Λ_{2x}, and the longitudinal scale in the z direction (Λ_{3z}) is only about half that in $x(\Lambda_{1x})$. When the model results are viewed, it should be remembered that they are obtained by prescribing only a single length scale value and that all other variations are obtained implicitly from the Reynolds stress profile and continuity requirement. There are differences with measurements, specifically, the model substantially overpredicts Λ_{3z} (giving a value only a little lower than Λ_{1x}) and Λ_{1z}. However, there are also similarities. The model appears to capture some of the anisotropy in the u_2 length scales (correctly predicting $\Lambda_{2z} > \Lambda_{2x}$) and the shapes of the Λ_{3x} and Λ_{1z} profiles.

Characteristic Eddy Decomposition

It is tempting to use pictures such as Fig. 2 to speculate about the form of the instantaneous eddy structures responsible for the

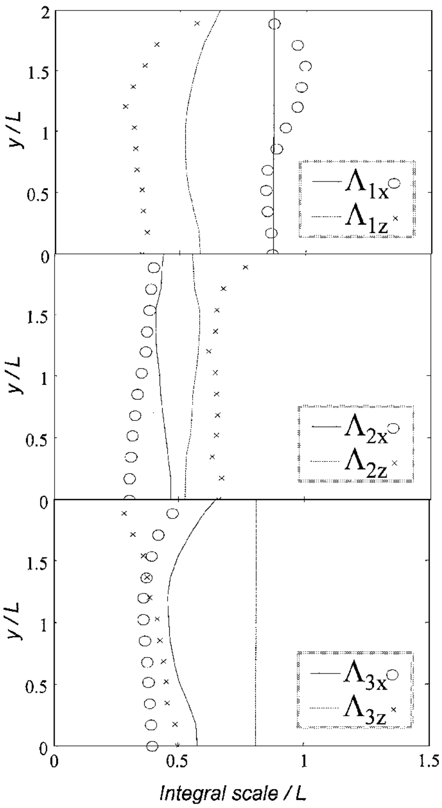

Fig. 4 Profiles of streamwise and spanwise integral scales: points are measurements and lines are model.

measured correlation maps. However, one must remember that Fig. 2 shows only one cut through a four-dimensional function and, even at the resolution of the measured data, one could plot some 10,000 similar figures for all of the possible combinations of $z'-z$ and τ. One therefore seeks a more objective and efficient method for extracting physical information about the instantaneous flow.

Linear stochastic estimation (LSE) and proper orthogonal decomposition (POD) are both methods for extracting eddylike velocity fields from correlation functions. LSE[14] gives the best linear estimate of the instantaneous velocity field given some predetermined condition, such as the value of a velocity component at one point $u_i(y)$. In this case, the estimated velocity field is simply given by the two-point correlation function itself,

$$u_j(y', z'-z, \tau)\,|_{\mathrm{LSE}} = R_{ij}(y, y', z'-z, \tau)\left[u_i(y)\big/\overline{u_i(y)^2}\right] \quad (11)$$

(no summation implied).

POD[15] provides a means to compute the optimum basis of the instantaneous velocity field, that is, a series of orthogonal functions, or modes, that on average provide the best fit to the instantaneous velocity field. If the instantaneous flow can be characterized as a superposition of frequently appearing eddy types, then one might reasonably expect the modes produced by POD to represent the form of those eddies. By maximizing the correlation with the instantaneous velocity field, Lumley[15] shows that the modes are eigenfunctions of the two-point correlation tensor and that their spectrum is given by the corresponding eigenvalues. POD appears to work well in inhomogeneous directions, such as the y direction in the present flow. One can, for example, perform a one-dimensional POD of the wake by solving the Fredholm integral (with summation)

$$\int R_{ij}(y, y', 0, 0)\phi_j(y')\,\mathrm{d}y' = \lambda\phi_i(y) \quad (12)$$

This integral equation has multiple eigenfunction solutions $\phi_i^{(n)}(y)$ representing the orthogonal modes of the instantaneous wake velocity profile and eigenvalues $\lambda^{(n)}$ (the spectrum) that reveal the proportion of the turbulence kinetic energy produced by each mode.

POD does not work as well in homogeneous directions because here it reduces to Fourier decomposition, and sinusoids are obviously not good representations of eddies. The three-dimensional POD of the wake flow involves Fourier transforming with respect to z' and τ (or x) and then solution of the integral,

$$\int R_{ij}(y, y', k_z, k_x)\phi_j(y', k_z, k_x)\,\mathrm{d}y' = \lambda(k_z, k_x)\phi_i(y, k_z, k_x) \quad (13)$$

where k_z is spanwise wave number and k_x represents frequency or streamwise wave number. Again there are multiple solutions, but each is multiplied by sinusoidal variations in the homogeneous directions. The usual way to construct a three dimensionally compact representation of a characteristic eddy from these modes[16] is to extract only the dominant mode at each wave number combination and then to inverse Fourier transform the result, making assumptions about the relative phasing of these modes. One problem with this approach is that it appears to assume a correlation between the dominant modes, when in fact the POD implies the absence of any such correlation.

An alternative approach, which we employ here because of its value to aeroacoustics[17–19] is to use POD only to extract the modal profiles in the inhomogeneous direction, according to Eq. (12). We then use LSE to obtain the best linear estimate of the complete three-dimensional instantaneous velocity field associated with each mode. For the nth mode $\phi_i^{(n)}(y)$ we obtain

$$u_j^{(n)}(y', z'-z, \tau)\,|_{\mathrm{CES}} = \frac{1}{\lambda^{(n)}}\int \phi_i^{(n)}(y)R_{ij}(y, y', z'-z, \tau)\,\mathrm{d}y \quad (14)$$

The resulting three-dimensional modes are termed compact eddy structures (CES).[17,18] Just as with LSE and the characteristic eddy of the three-dimensional POD, the extent to which compact eddy structures are representative of the typical form of the instantaneous motions present in the turbulence is a matter for debate. However, unlike the alternatives, the compact eddy structures defined by Eq. (14) also provide an efficient and complete basis for reconstructing the correlation function. This means, for example, that the broadband noise generated by the intersection of a lifting surface and an inhomogeneous turbulent flow can be decomposed into calculations of the noise generated by individual compact eddy structures.[17,18] Indeed, unlike the more conventional approach of describing the turbulence in terms of a wave number frequency spectrum, such a methodology is rigorously consistent with the inhomogeneity of the flow. Glegg and Devenport[18,19] show that for a wake flow only the few most energetic compact eddy structures are needed to capture the bulk of the radiated noise.

Figure 5 shows the eigenvalue spectrum from the solution to Eq. (12) and Fig. 6 shows the first four modal vector profiles. The measurement resolution allowed for the calculation of about 100 modes, of which the first 40 appeared free of any significant aliasing effects. Figures 7–9 show the three-dimensional velocity fields of the compact eddy structures corresponding to these four modes. Results have been computed both from the measured correlation

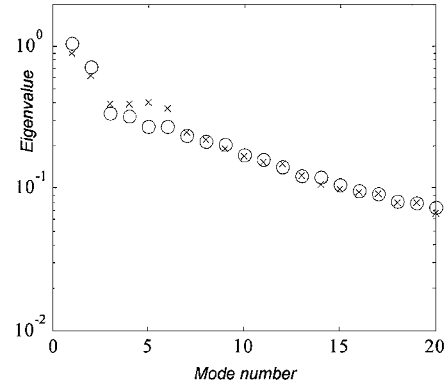

Fig. 5 Eigenvalue spectra from the one-dimensional proper orthogonal decomposition in the y direction.

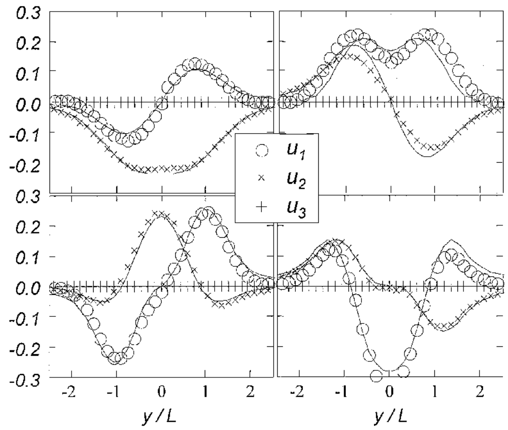

Fig. 6 Model profiles for the first four modes: symbols show measurements, and lines show model results.

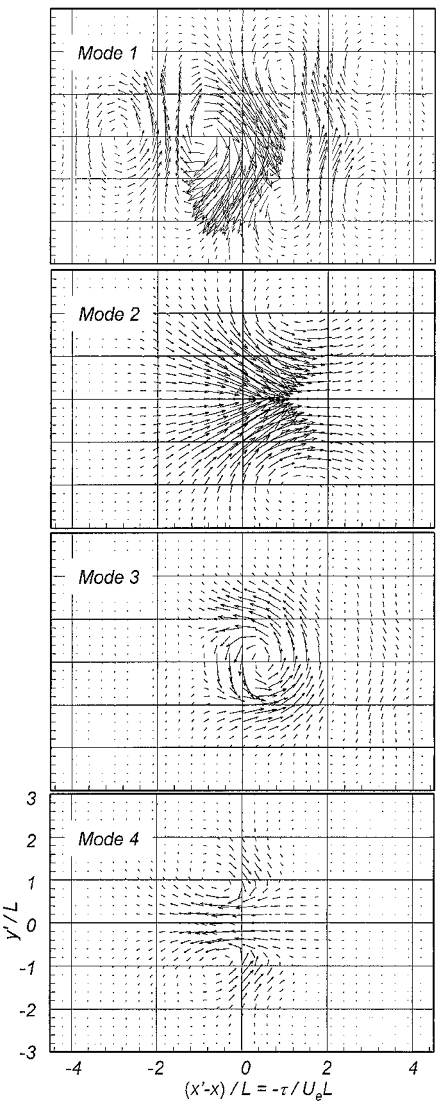

Fig. 7 Compact eddy structures for the first four modes deduced from the measured correlation function.

tensor function and from the model function extrapolated from the Reynolds stress profiles.

The measured eigenvalue spectrum (Fig. 5) shows that the first two modes contain significantly more energy than the others. Together these two modes account for about 27% of the total turbulence kinetic energy in the wake. The corresponding modal profiles (Fig. 6) are clearly associated with the generation of the Reynolds shear stress because they combine symmetric and antisymmetric u_1

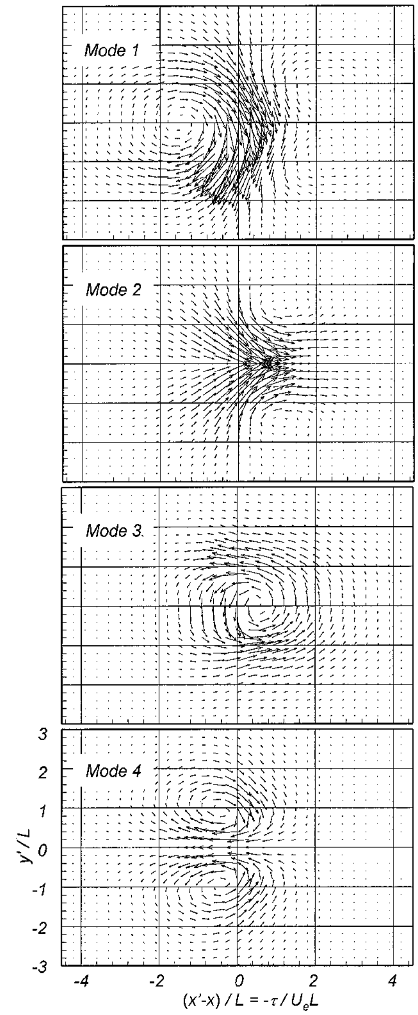

Fig. 8 Compact eddy structures for the first four modes deduced from the model correlation function.

and u_2 profiles. The third and fourth modes appear to imply more complex motions (u_3 is negligible in the first four modes). The $z = 0$ cuts through the corresponding compact eddies (Fig. 7) show predominantly spanwise roller-type structures appearing either singly (modes 1 and 3) or in symmetric pairs (modes 1, 2, and 4). There is, of course, three-dimensional structure associated with these cuts. In particular, the form of the mode 2 eddy away from the $z = 0$ plane (Fig. 9) bears some resemblance to the so-called double-roller eddy structure first identified as a significant feature of plane wake turbulence by Payne and Lumley.[20]

One would expect the model correlation tensor function to show few if any of these features because the model contains no physics beyond the Reynolds stress profiles and a single length scale. It is, therefore, intriguing that the modal spectrum (Fig. 5) obtained from the model is very similar to that measured. [The biggest differences are an underprediction (of between 10 and 20%) of the eigenvalues of the first two modes, and a similar overprediction of the eigenvalues of modes 3–6.] In addition, the model seems to provide an accurate prediction of the modal profiles (Fig. 6) and also reproduces the dominant features of all of the associated three-dimensional characteristic eddy structures (Figs. 8 and 9). (Plots like Fig. 9 for the other modes show a similar level of agreement between measurement and model.) Estimates of the three-dimensional velocity fields associated with the wake eddies based on LSE alone [through application of Eq. (11)] show similar agreement between model and measurement.

These results present an interesting choice. One could interpret them either as demonstrating that the overall instantaneous form of the coherent structures in the wake can be predicted merely from the Reynolds stress distribution and a length scale, or (more likely) that characteristic eddy structures inferred from the measured

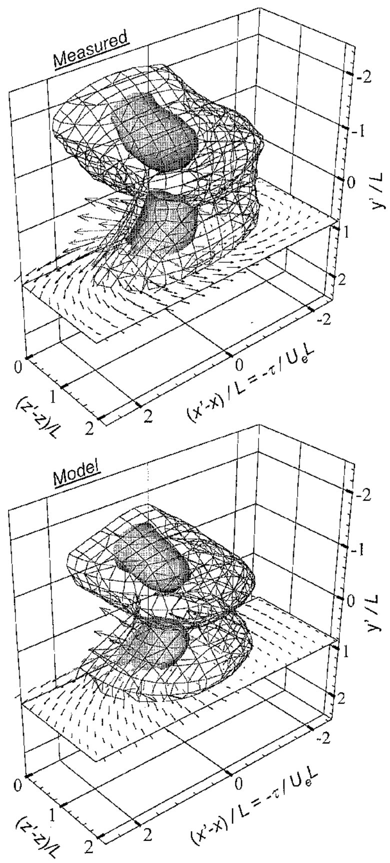

Fig. 9 Three-dimensional form of the mode 2 compact eddy structure: isosurfaces show vorticity magnitudes (normalized on U_w and L) of 0.05 and 0.13.

correlations contain little information beyond that implied by the Reynolds stress profile and a single length scale. Whatever the case, it appears that the correlation function, its modes, and their associated velocity fields can, at least approximately, be predicted from single-point turbulence information. This implies substantial simplification to the problem of specifying two-point turbulence correlations for aeroacoustics because single-point turbulence information and length scales can be obtained from computational fluid dynamics (CFD) solutions.

Finally, we wish to comment on the model calculations being performed using a single constant length scale. By doing the calculations in this way, we have tested the basic hypothesis of the model, that is, we have demonstrated the extent to which the model can predict local features of the correlation function when that local information is provided solely through the Reynolds stress field. Incorporation of more refined empirical input, such as variable or anisotropic length scale and/or decay function is certainly feasible[13] and would likely lead to improvements in accuracy. However, we leave this as a topic for further research.

Conclusions

1) The four-dimensional two-point correlation tensor function of the fully developed turbulent wake of an airfoil has been measured.

2) A compact representation of this function, suitable for use in aeroacoustic calculations, has been extracted from these measurements.

3) A simple generic technique has been developed to extrapolate this information from the Reynolds stress field. Applied to the wake, this reproduces many of the features of the correlation function, including the dominant proper orthogonal modes, their spectrum, and associated characteristic eddy structures.

4) This implies that the local features of the wake correlation function can in large part be predicted from the local features of the Reynolds stress field.

5) This also implies that characteristic eddy structures inferred from the measured two-point correlation function contain little physical information about the instantaneous eddies beyond that which can be inferred from the Reynolds stress profile and a single length scale.

6) The similarity in characteristic eddy structures inferred from the measured and model correlation functions implies substantial simplification to the problem of specifying two-point turbulence correlations for aeroacoustics because it suggests that such correlations could be estimated using the type of turbulence information available from CFD solutions.

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References

[1] Amiet, R. K., "Acoustic Radiation from an Airfoil in a Turbulent Stream," *Journal of Sound and Vibration*, Vol. 41, No. 4, 1975, pp. 407–420.

[2] Glegg, S. A. L., Wittmer, K. S., Devenport, W. J., and Pope, D. S., "Broadband Noise Generated by Blade Wake Interactions," *Journal of the American Helicopter Society*, Vol. 44, No. 4, 1999, pp. 293–301.

[3] Devenport, W. J., Wenger, C. W., Glegg, S. A. L., and Miranda, J. A., "Wave Number Frequency Spectra of a Lifting Wake for Broadband Noise Prediction," *AIAA Journal*, Vol. 36, No. 6, 1998, pp. 881–887.

[4] Wittmer, K. S., Devenport, W. J., and Wenger, C. W., "Turbulence Structure Resulting from a Vortex/Blade Tip Interaction," *Journal of the American Helicopter Society*, Vol. 44, No. 2, 1999, pp. 141–149.

[5] Wenger, C. W., Devenport, W. J., Wittmer, K. S., and Muthanna, C., "Two-Point Measurements in the Wake of a Compressor Cascade," AIAA Paper 98-2556, 1998.

[6] Wittmer, K. S., Devenport, W. J., and Zsoldos, J. S., "A Four-Sensor Hot-Wire Probe System for Three-Component Velocity Measurement," *Experiments in Fluids*, Vol. 24, Nos. 5–6, 1998, pp. 416–423.

[7] Wittmer, K. S., Devenport, W. J., and Zsoldos, J. S., "A Four-Sensor Hot-Wire Probe System for Three-Component Velocity Measurement," *Experiments in Fluids*, Vol. 27, No. 4, 1999, p. U1.

[8] Miranda, J., "The Structure of a Trailing Vortex Wake," M.S. Thesis, Dept. of Aerospace and Ocean Engineering, Virginia Polytechnic Inst. and State Univ., Blacksburg, VA, 1996.

[9] Kline, S. J., and McClintock, F. A., "Describing Uncertainties in Single Sample Experiments," *Mechanical Engineering*, Vol. 75, No. 1, 1953, p. 3.

[10] Devenport, W. J., Muthanna, C., Ma, R., and Glegg, S. A. L., "Measurement and Modeling of the Two-Point Correlation Tensor of a Plane Wake for Broadband Noise Prediction," Dept. of Aerospace and Ocean Engineering, Rept. VPI-AOE-263, Virginia Polytechnic Inst. and State Univ., Blacksburg, VA, 1999.

[11] Chandrasekhar, S., "The Theory of Axisymmetric Turbulence," *Philosophical Transactions of the Royal Society, Series A: Mathematical and Physical Sciences*, Vol. 242, 1950, pp. 557–577.

[12] Hinze, J. O., *Turbulence*, McGraw–Hill, New York, 1975.

[13] Devenport, W. J., and Glegg, S. A. L., "Modeling the Two-Point Space–Time Correlation of Turbulence in a Fan-Wake Type Flow," AIAA Paper 2001-2241, May 2001.

[14] Adrian, R. J., "Stochastic Estimation of the Structure of Turbulent Fields," *Eddy Structure Identification*, edited by J. P. Bonnet, Springer-Verlag, New York, 1996, pp. 145–195.

[15] Lumley, J., "The Structure of Inhomogeneous Turbulent Flows," *Atmospheric Turbulence and Radio Wave Propagation*, edited by A. M. Yaglom and V. I. Tatarsky, Nauka, Moscow, 1967, pp. 166–176.

[16] Berkooz, G., Holmes, P., and Lumley, J. L., "The Proper Orthogonal Decomposition in the Analysis of Turbulent Flows," *Annual Review of Fluid Mechanics*, Vol. 25, 1993, pp. 539–575.

[17] Glegg, S. A. L., and Devenport, W. J., "Compact Eddy Structures and Their Application to Aeroacoustics," AIAA Paper 2000-1995, June 2000.

[18] Glegg, S. A. L., and Devenport, W. J., "Proper Orthogonal Decomposition of Turbulent Flows for Aeroacoustic and Hydroacoustic Applications," *Journal of Sound and Vibration*, Vol. 239, No. 4, 2001, pp. 767–784.

[19] Glegg, S. A. L., and Devenport, W. J., "Rotor/Stator Interaction Noise from Blade Wakes," AIAA Paper 2001-2150, May 2001.

[20] Payne, F. R., and Lumley, J. L., "Large Eddy Structure in the Turbulent Wake Behind a Circular Cylinder," *Physics of Fluids*, Supplement, 1967, pp. S194–S196.

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